Fault pruning:
Robust training of neural networks with memristive weights

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Motivation

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  - Main features: neuron-synapse structure, in-memory computation, learning capabilities

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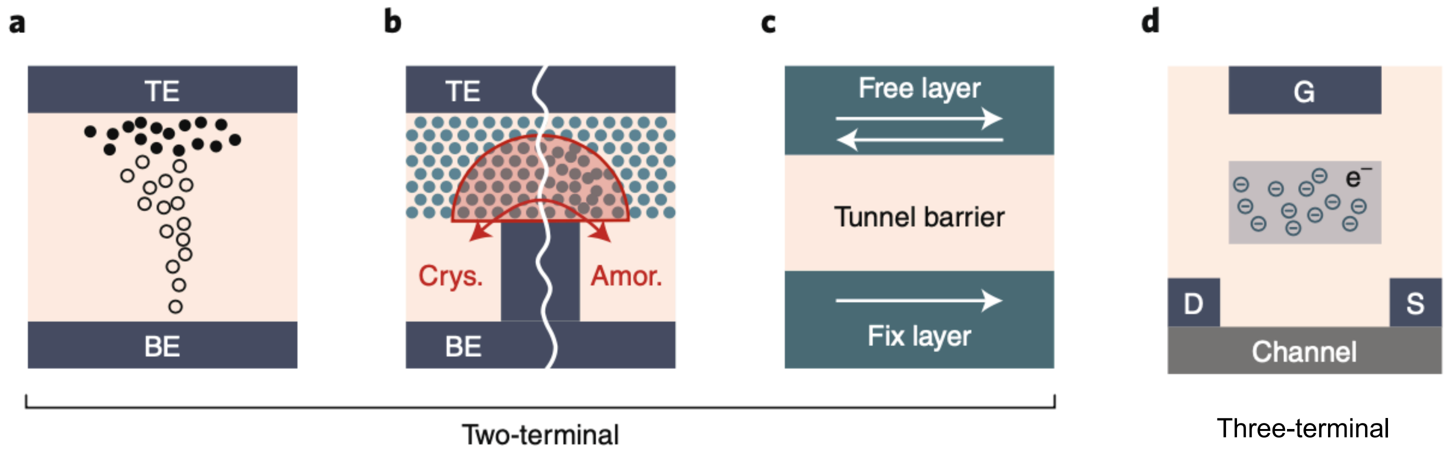

Image source:
Zhang, Wenqiang, et al. "Neuro-inspired computing chips." *Nature electronics* 3.7 (2020): 371-382.

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- **Key metrics** for performance evaluation:
  - Computing density
  - Energy-efficiency
  - Computing accuracy: influenced by non-idealities of devices
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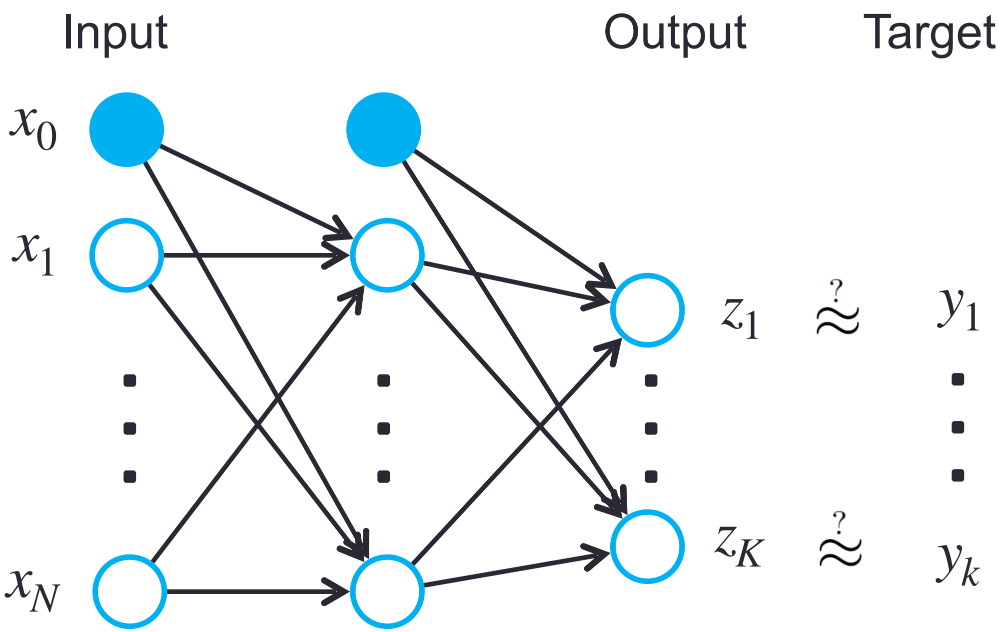
- **Our focus:**
  - RRAM devices (“**memristors**”)
  - Improving energy-efficiency
  - Learning “in-the-loop”:
    - robust training of neural networks with memristive weights
    - detection of faulty memristors
    - improving computing accuracy

Introduction: Neural networks

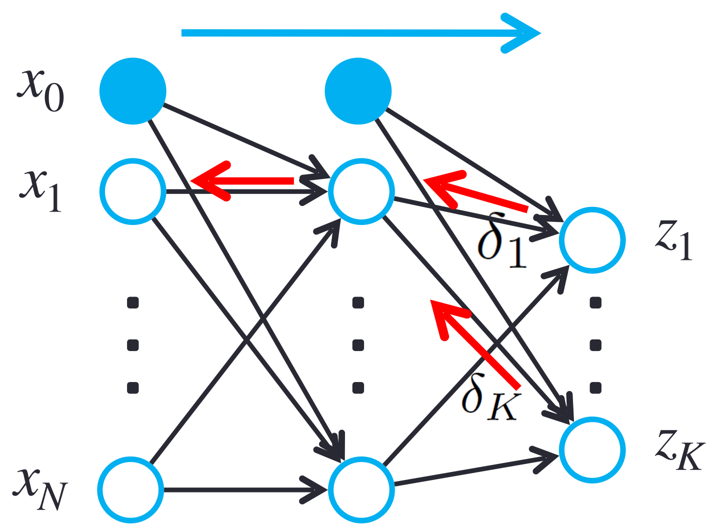


Learning:
the weights are optimized by minimizing the training error.

e.g., $E = \frac{1}{2}\sum_{k=1}^{K} e_k^2 = \frac{1}{2}\sum_{k=1}^{K} (z_k - y_k)^2$

Introduction: Neural networks

- For learning, the gradient of the error function is needed.
  - Forward: Calculate activations and outputs of all neurons.
  - Backward: Calculate errors and propagate them back



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- Mimic biological synapses
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- Can operate in an analog fashion
- Suited for implementation of matrix-vector multiplications
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Challenges:

- Fabrication, operational constraints
- Limited endurance of the devices
- Yield and repeatability issues

Memristive neural network training

Faulty behavior of memristors

- Stuck memristors
- Faulty updates
  - Concordant switching faults
  - Discordant switching faults

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Our approach:

- Analyze impact of faulty memristor behavior on neural network training
- Strategy: Use Fault pruning.
  Detection of faults during training and pruning of connections on the fly.

Memristive weights

- Mapping resistance $R_i \in [R_{\text{min}}, R_{\text{max}}]$ to weight $w_i \in [w_{\text{min}}, w_{\text{max}}]$:

$$w_i = \alpha\left(\frac{1}{R_i} - \frac{1}{R_C}\right)$$

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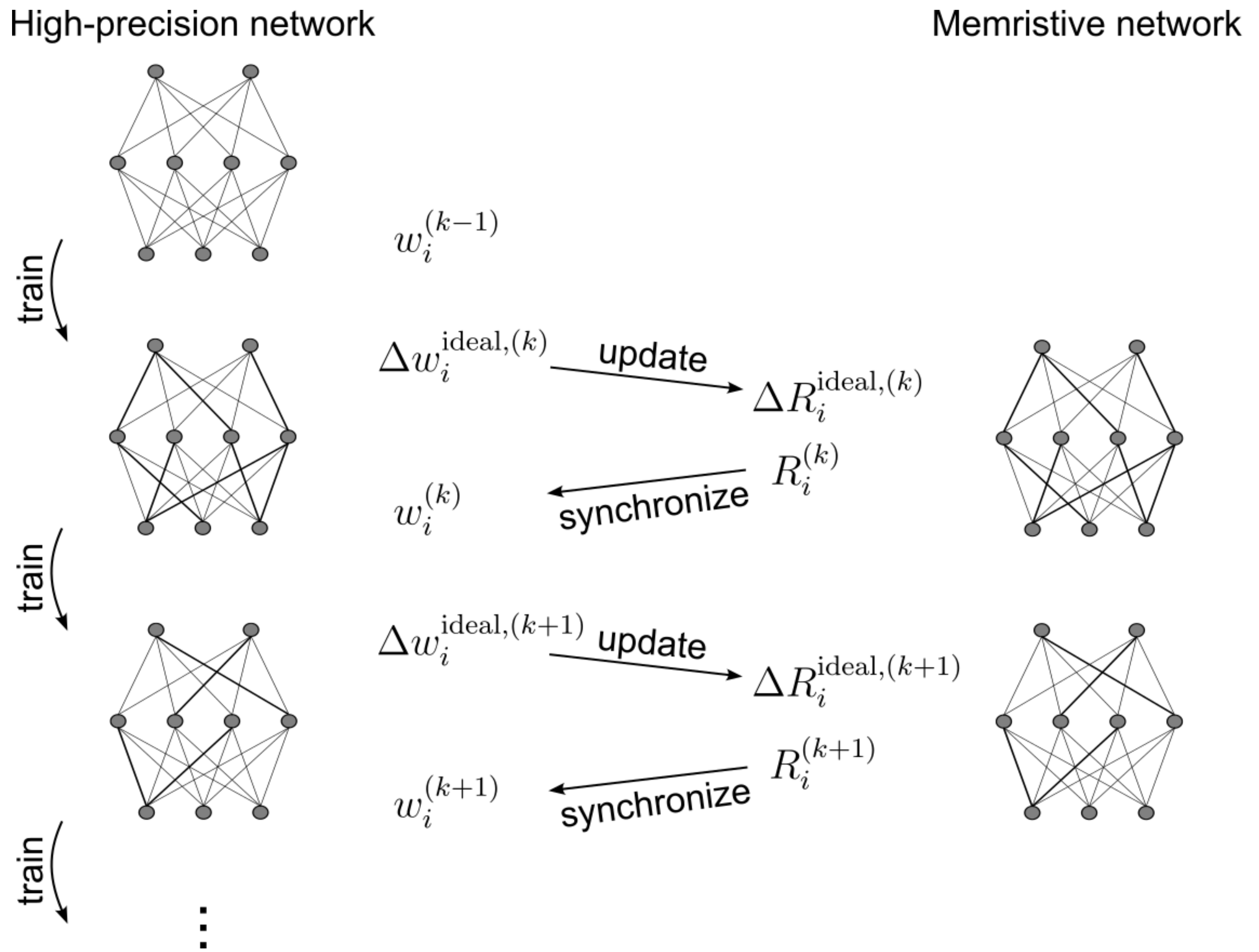
- Weight and resistance updates:

$$\Delta w_i = w_i^{(k)} - w_i^{(k-1)}$$
$$\Delta R_i = R_i^{(k)} - R_i^{(k-1)}$$

In-the-loop training



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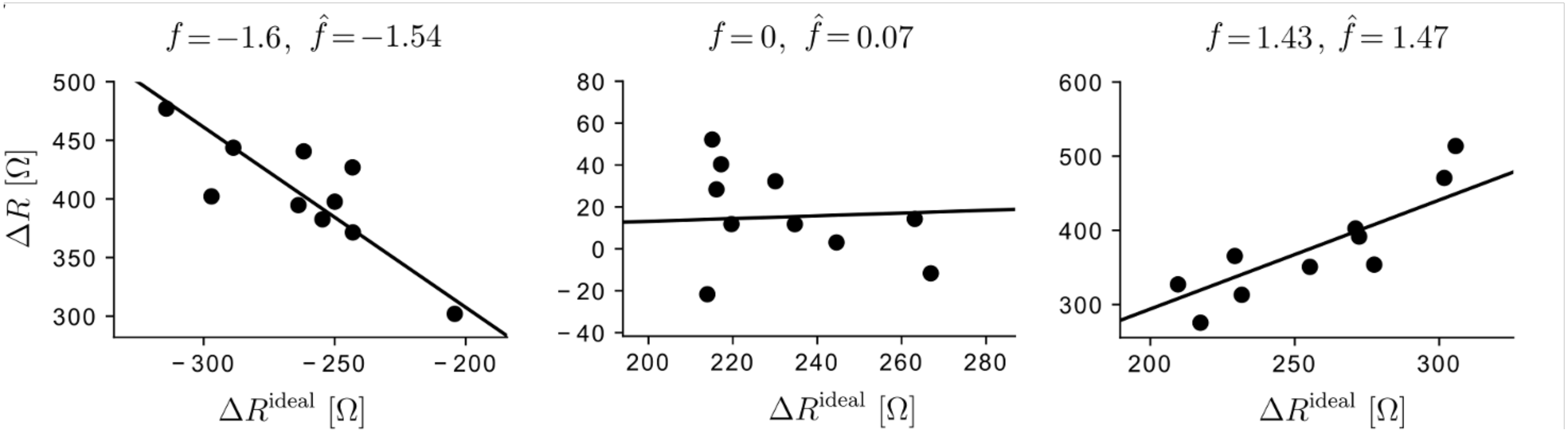
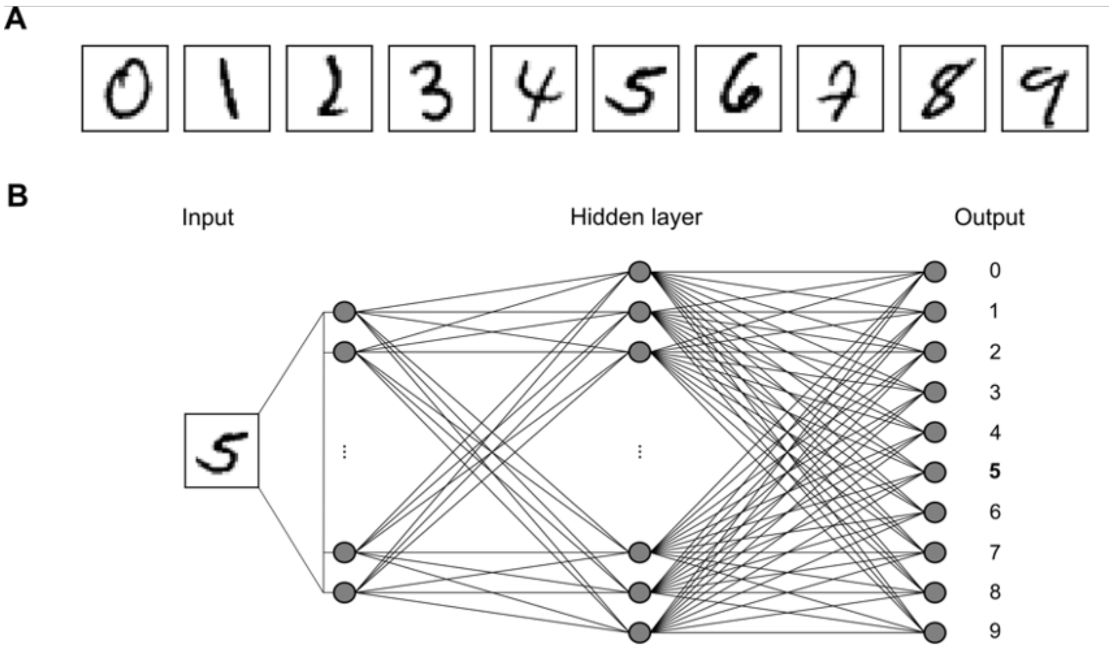
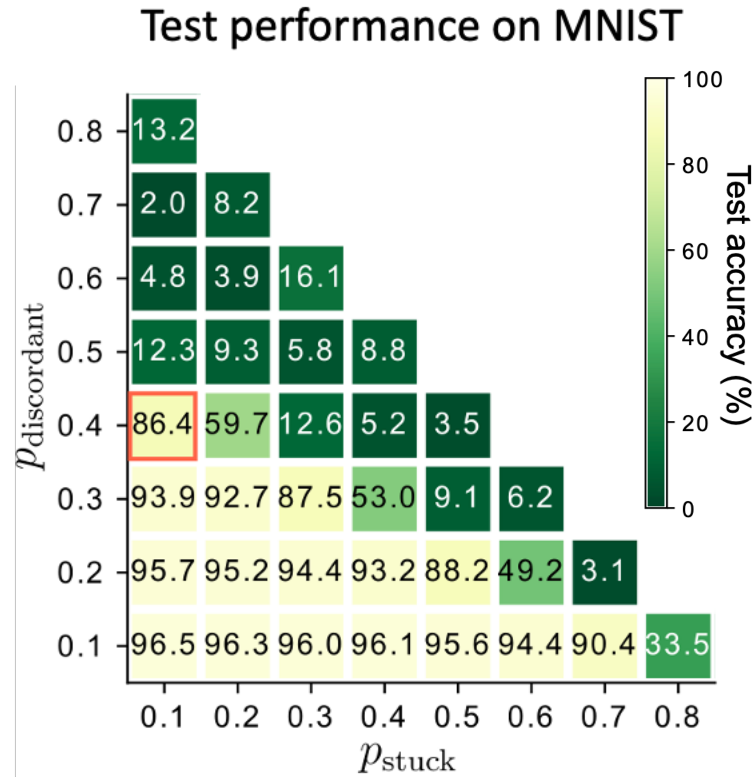
Model of imperfect memristor

- Memristor faults modeled by fault factor f_i
  - Modulates memristance change: $\Delta R_i^{(k)} = f_i \cdot \Delta R_i^{\text{ideal}, (k)} + \eta_i^{(k)}$
  - Stuck memristors: $f_i = 0$
  - Concordant changes: $f_i > 0$
  - Discordant changes: $f_i < 0$
  - Switching and readout noise $\eta_i^{(k)}$ added.



The MNIST task





Discordant memristive changes are detrimental.

Neural networks can be pruned significantly and achieve little loss in accuracy, hence we asked if one can prune faulty memristive connections.

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Fault pruning algorithm

- Estimate fault factor over a window of previous updates:

$$\hat{f}_i = \frac{\sum_l \Delta R_i^{\text{ideal},(l)} \Delta R_i^{(l)}}{\sum_l \left(\Delta R_i^{\text{ideal},(l)}\right)^2}$$

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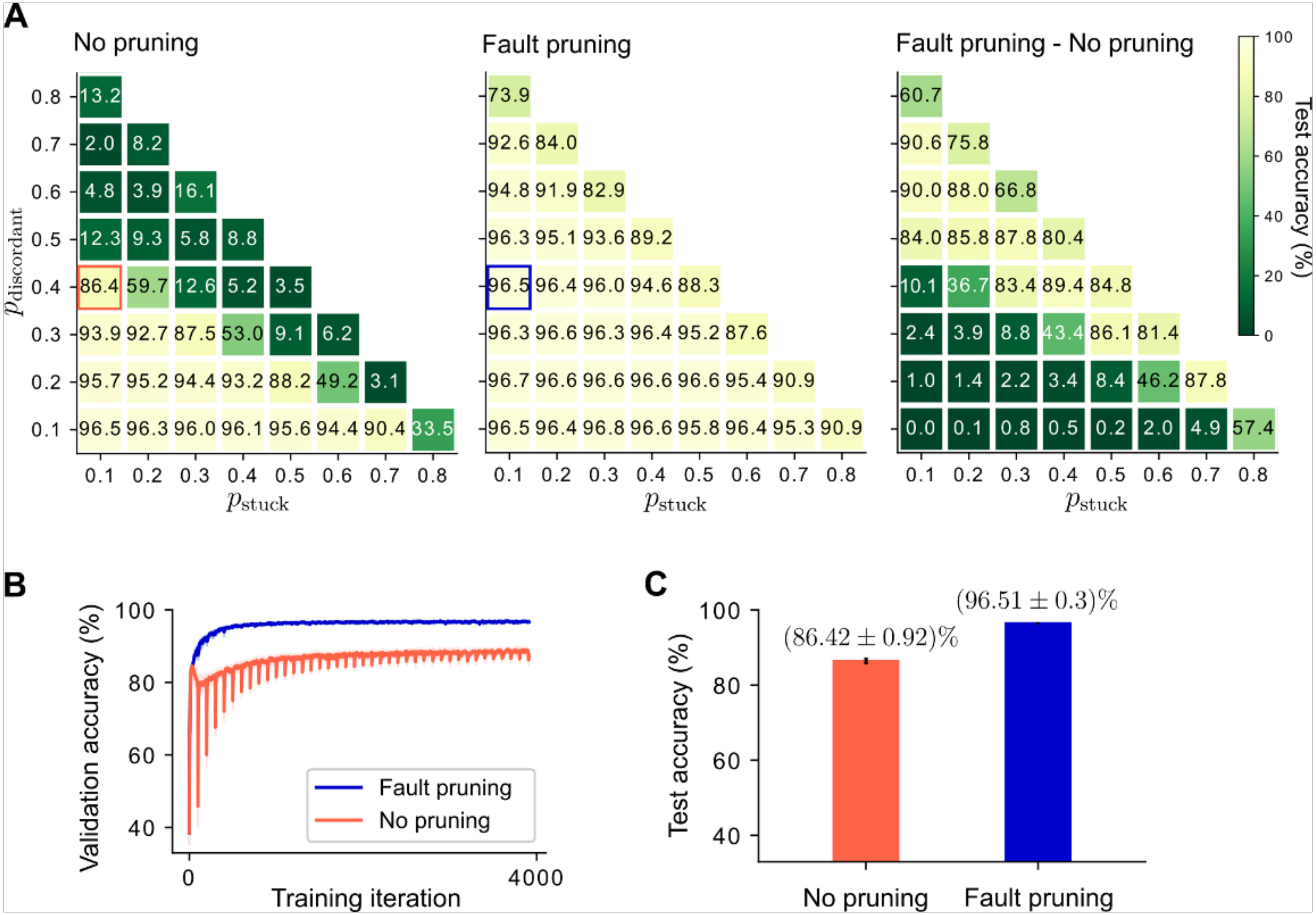
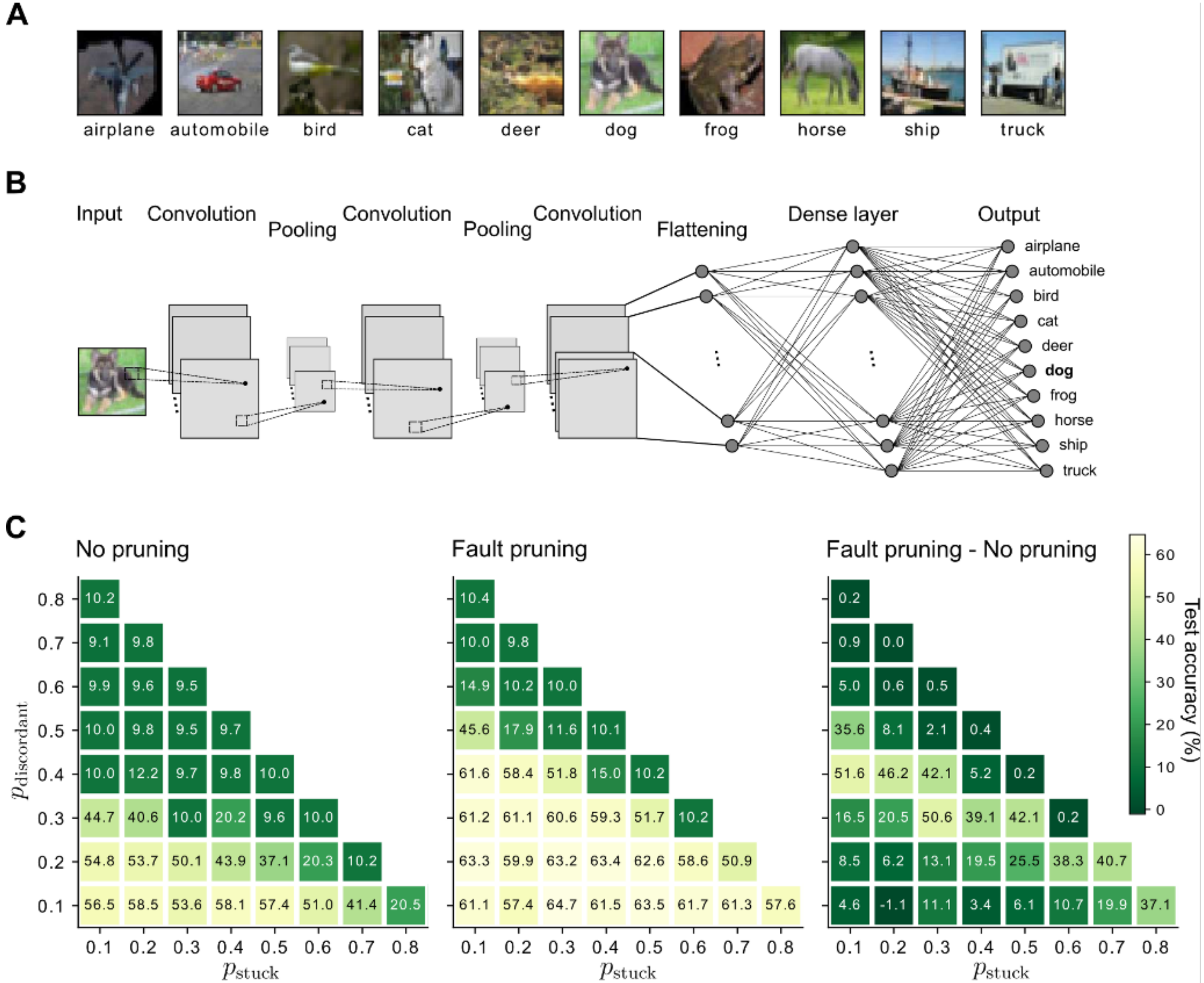
- Remove detected unreliable memristors from the network if $\hat{f}_i < \theta$, and we set $\theta = 0.1$

- Two variants of the algorithm
  - Variant 1: Prune faulty weights (set to zero)
  - Variant 2: Don't update faulty weights (keep last weight)

Results on MNIST



Results on CIFAR-10



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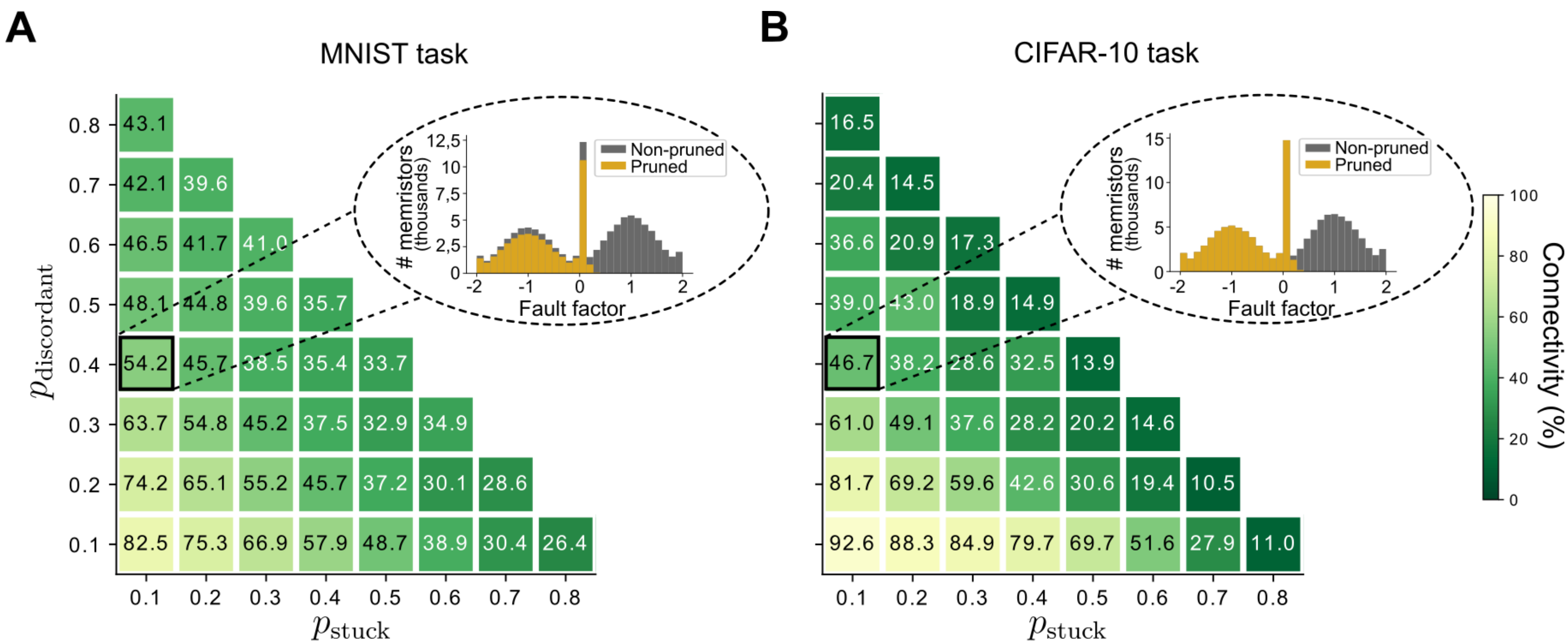
Connectivity in the network after pruning



Summary and Conclusion

- Fault pruning managed to preserve very good performance
- Estimation of faults on the fly, and acting accordingly
- General approach, independent of the network structure and trained tasks
- A simple linear regression to estimate faults
  - Can be substituted by more advanced approaches
- **Future work:**
  - Test the algorithm in a real-world scenario
  - Handling memristors with discordant faults - by adapting the requested update

Questions?

Estimation of the fault factors $\hat{f}_i$

$$\Delta R_i = \hat{f}_i \cdot \Delta R_i^{\text{ideal}} + \epsilon$$

Estimated from $N = 10$ data points $(\Delta R_i^{\text{ideal},(l)}, \Delta R_i^{(l)})$, $l \in \{k-N+1, k-N+2, \ldots, k-1, k\}$

The least-squares estimator of $\hat{f}_i$ minimises the error

$$\mathscr{L}(\hat{f}_i) := \sum_l \left(\Delta R_i^{(l)} - \hat{f}_i \cdot \Delta R_i^{\text{ideal},(l)}\right)^2,$$

$$\frac{\partial \mathscr{L}}{\partial \hat{f}_i} = 2 \sum_l \left(\Delta R_i^{(l)} - \hat{f}_i \cdot \Delta R_i^{\text{ideal},(l)}\right)\left(-\Delta R_i^{\text{ideal},(l)}\right) \stackrel{!}{=} 0$$

$$\hat{f}_i \sum_l \left(\Delta R_i^{\text{ideal},(l)}\right)^2 = \sum_l \Delta R_i^{(l)} \Delta R_i^{\text{ideal},(l)}$$

$$\hat{f}_i = \frac{\sum_l \Delta R_i^{(l)} \Delta R_i^{\text{ideal},(l)}}{\sum_l \left(\Delta R_i^{\text{ideal},(l)}\right)^2}$$